
python-with-science

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This is an introduction and some projects for beginners in Python. The projects use turtle graphics so results are visually pleasing from the start.

They were written for an after-school coding club at junior level (years 4 to 6). The projects are challenging for this age-group, but the club is strongly-supported by knowledgeable helpers.

The introductory two modules are adapted for the Raspberry Pi, but could easily be interpreted for any computer running Python 3. The project modules themselves (modules 3 and up) do not depend at all on the type of computer in use.

... WITH SOME SCIENCE

These projects can be taken purely as exercises in coding and debugging. However, each project connects to science and mathematics (STEM) topics outside the coding: shapes in nature, colour perception and mathematical curves.

Many coding projects for young people focus on the creation of a game, as if this were the only use of a computer likely to interest them. The author has never grown out of play, but his view is that the real world is more interesting than any game.

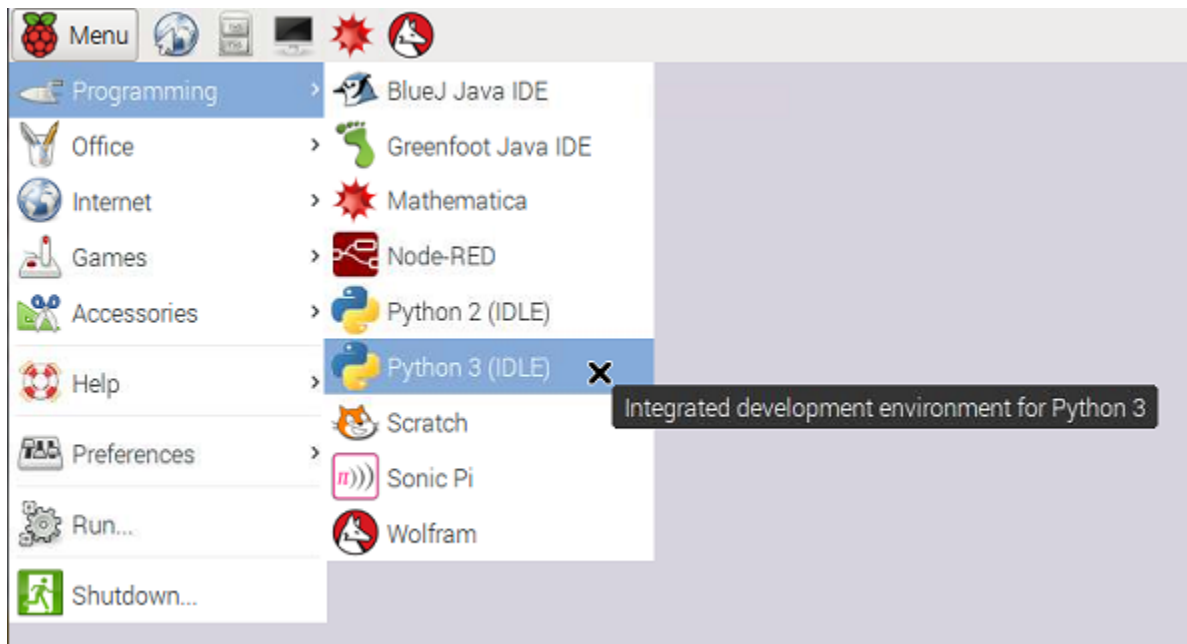
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2.1 The shell: awesome calculator

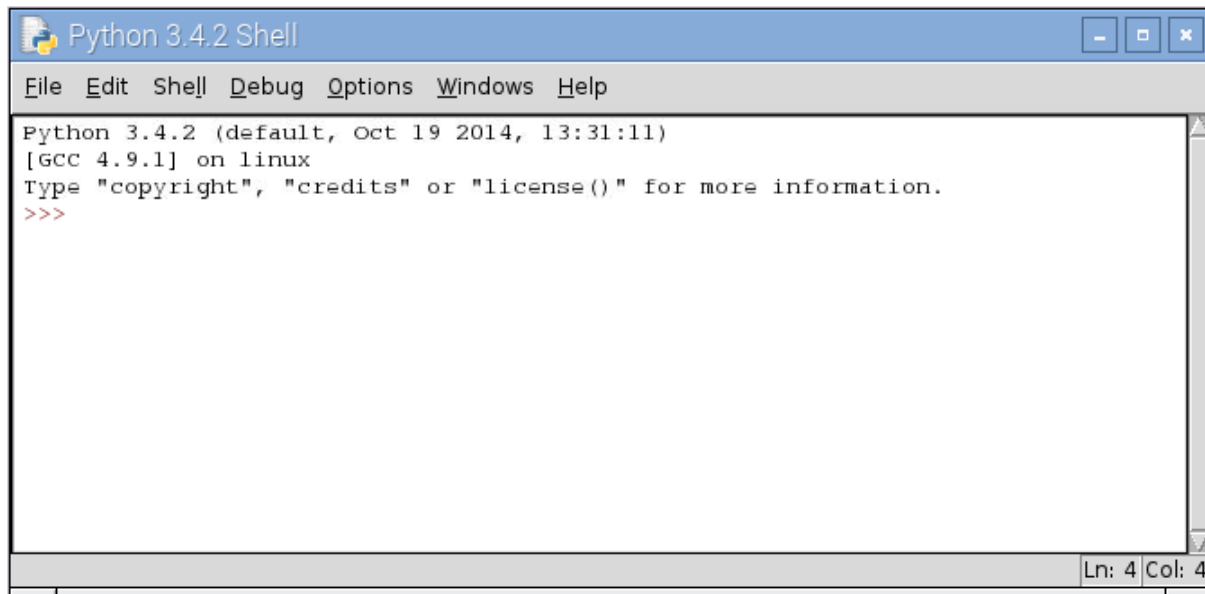
We can talk directly to Python through a program called “the shell”. You can run the shell by launching IDLE.

2.1.1 Launching IDLE

Launch IDLE from the “programming” section of the desktop menu. The menu will look a little like this:



Make sure you pick IDLE 3. This will open a “shell” window like this:



Python is ready to do things for you. At the `>>>` prompt type `6*7`. The star `*` means multiplication. Python will reply with the answer 42.

Hint: You will often make mistakes and get rude, red error messages from Python. Don't worry: this is normal in programming. Try to guess what you did wrong and try again at the next `>>>` prompt.

Now try this:

```
>>> a = 7
>>> b = 6
>>> a * b
42
```

`a` and `b` are *variables*. A variable is just a name you use for a value that matters to you. The `=` means “make the name refer to this value”.

About names for variables

You can use almost name for a variable, made of letters, digits and the `_` character, but not spaces. An example is `top_5_cats`. You can't start with a digit, because that looks like a number to Python. You can't use the 33 words already taken by Python (like `for` and `in`), that come out orange in IDLE. You *can* use names like `print` or `list` (purple in IDLE) that already mean something to Python, but it confuses humans.

Programmers use short names (letters) for variables that are not needed for long. When a variable will be used in lots of places, they use long helpful names to remind them what it means.

When we tell Python to do a calculation, instead of a number, we can use a variable that refers to the number. It could be an easy calculation, or one that's a bit complicated:

```
>>> a + b
13
>>> (a+b) * (a-b)
13
```

You can make the variable (the name) mean something different any time you like. Try:

```
>>> a = 116
```

Let's re-calculate the complicated expression, but without typing. In the IDLE shell, you can re-use lines you typed earlier. Put the cursor on the line that says `(a+b) * (a-b)`, using the mouse or the arrow keys, and press enter. This makes a copy of that line at the prompt, as if you had typed it. Press enter again, and Python will work it out:

```
>>> (a+b) * (a-b)
13420
```

This time, the meaning of `a` is different, so you get a different answer.

A variable can refer to any kind of information. Sometimes you want to refer to a *string* of text. Strings are written in quotes. Single and double quotes mean the same, and you can use `+` to stick strings together:

```
>>> h = 'Hello'
>>> w = "world!"
>>> h + ' ' + w
'Hello world!'
```

2.1.2 Again, again!

Just now, we used the same expression over again with a new value by copying. Often, you need to work something out for a whole list of values. Here is a list being assigned to a variable and printed out (try it):

```
>>> alist = [2, 0.5, a, 2*b, "banana"]
>>> print(alist)
[2, 0.5, 116, 12, 'banana']
```

Notice that the expressions in the list (like `a` and `2*b`) were calculated as the list was being made.

Now type at the prompt:

```
>>> n = 3
>>> for x in alist:
    print(n, 'times', x, 'is', n*x)
```

After the first line, the IDLE shell will move the cursor across, so the next line starts in the right place. It will do that again after your print statement in case you have another line to enter. Enter an empty line and it will know you've finished.

If you made a mistake, use the trick you learned earlier to copy your first attempt to the prompt so you can edit it. Put the cursor at the end of the last line and press enter twice to try again.

What happened here?

Python looked at `for x in` and understood that you wanted to use a *variable* called `x`, and make it refer to each of the values in the list. And for each different value given to `x`, you wanted to do what it says on the next line. The `print(...)` part means print those things on a line in the shell window.

We call this whole thing a “for-loop”, because the computer goes round and round the same instructions, once *for* each value.

Often we want to do this with a list of numbers. Remember you can save typing using the copy trick, just change `alist` to `range(13)` and hit return on the last line:

```
>>> for x in range(13):  
    print(n, 'times', x, 'is', n*x)
```

If you did it right, Python will have replied with the 3-times table. Or the times-table for whatever you set `n` to. Try the biggest number you can think of.

The `range(13)` that comes where `alist` used to be, means “all the numbers from zero up to, but not including, 13”. You can see the lists `range` makes like this:

```
>>> list( range(10) )  
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9]  
>>> list( range(5,13) )  
[5, 6, 7, 8, 9, 10, 11, 12]
```

2.1.3 Space is big ... mind-bogglingly big

Super powers of the mathematician

A mathematician would call these “the powers of 3”. She would write them like this: 3^4 , 3^5 and 3^6 , and say them like “3 to the 4” and so on.

Notice that the order of the numbers matters when you compute powers:

```
>>> 6**10  
60466176  
>>> 10**6  
1000000
```

6^{10} is a much bigger number than 10^6 (a million).

Try some really big powers. What are 3^{100} , 3^{1000} , and 3^{10000} ? What do you notice about the last few digits of these numbers? And do you get the same pattern with powers of other numbers (7 or 2, say)? Patterns in powers of numbers are behind the codes that keep us safe on the Internet.

If you want to investigate this, Python will do the hard work for you. (Use what you learned about the for-loop to make tables of powers.)

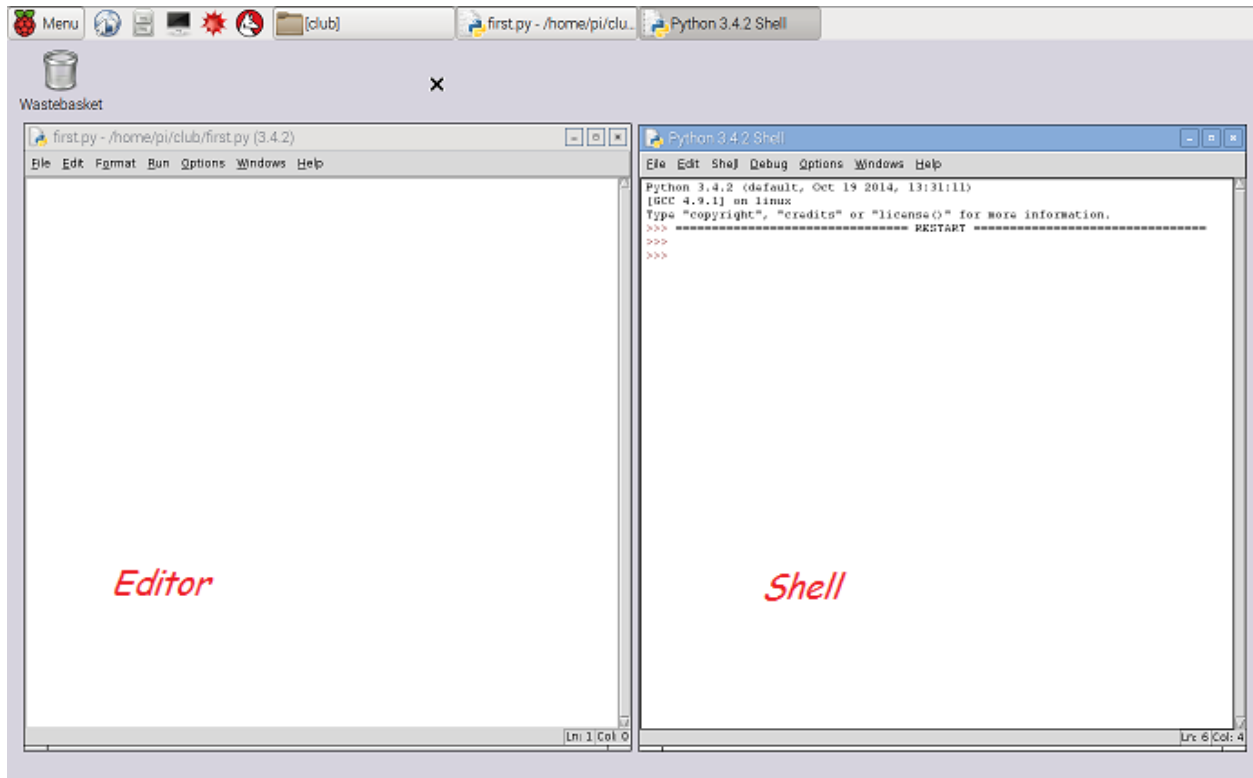
Sometimes we need to multiply the same number together several times:

```
>>> 3*3*3*3  
81
```

You *could* do that with a for-loop, but there’s a helpful shortcut for it:

```
>>> 3**4  
81  
>>> 3**5  
243  
>>> 3**6  
729
```

Python will happily calculate enormous numbers. Eighty tens multiplied together is roughly the number of atoms in the Universe:



Arrange the editor window on the left and the shell window in the right, if you want your work to match this guide. You can arrange them any way you want later.

2.2.2 Using the turtle library

We're going to draw some shapes using the `turtle` library. Type the following **in the editor** (left) window:

```
from turtle import *
speed("slowest")
forward(100)
```

Your program tells Python to bring all the things in the `turtle` library into your program. The `*` means “all” when it appears in an import statement. One of those things is the `speed` function; another is the `forward` function, but it is easiest to see what that does by running the program.

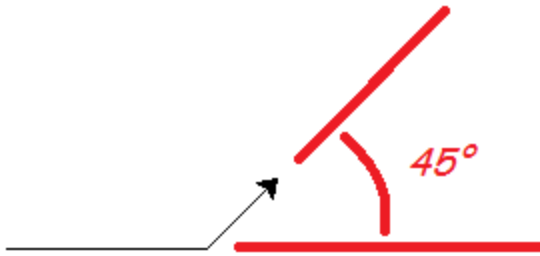
Save this, using the “File” menu, or by pressing `ctrl-S`. Now, run this program with `F5`. A third window opens and Python draws a line in it. This line is 100 units long (100 pixels). The arrowhead sitting at the end is the “turtle”, and wherever it goes, unless you tell it not to, it draws a line.



In the shell window (right), try a few more commands:

```
>>> left(45)
>>> forward(50)
```

`left(45)` means “turn left 45 degrees”. You will see that the turtle follows these commands exactly.



2.2.3 A triangle

Let's make a program to draw a triangle with 3 equal sides. Change your program so it looks like this:

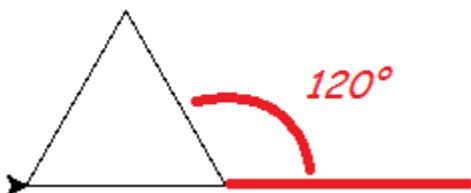
```
from turtle import *  
speed("slowest")  
  
a = 100  
  
forward(a)  
left(120)
```

Important: Make sure you type your program in the editor, not the shell window.

Then copy those last three lines and paste them so it looks like this:

```
from turtle import *  
speed("slowest")  
  
a = 100  
  
forward(a)  
left(120)  
  
forward(a)  
left(120)  
  
forward(a)  
left(120)
```

Save and run ... you drew a triangle!



This kind of cut-and-paste code is ugly to programmers. We do repetition with a *for-loop*. **Change** your code to be more elegant, like this:

```
from turtle import *
speed("slowest")

a = 100

for i in range(3):
    forward(a)
    left(120)
```

Run it to make sure you get the same result.

You can change the size of the triangle by changing the value *a* refers to. But suppose we want to change the number of sides? Could we just change the 3 to a 4 to draw a square? We will get 4 lines, but the angle 120 degrees is only correct for a triangle.

We can work out what to do. Watch the turtle carefully as it draws a triangle. It turns all the way round once, as if you had typed `left(360)` in the shell. (There are 360 degrees in a full rotation.) To get the shape we want, this rotation has to be shared equally between all the corners. So now we know how much to turn each time we draw a line. In a shape with *n* corners, it's $360/n$ degrees:

```
from turtle import *
speed("slowest")

a = 100
n = 4

for i in range(n):
    forward(a)
    left(360/n)
```

Try changing the value *n* refers to and running the program again.

2.2.4 Define your own function

You have been happily using functions like `range`, `print` and `forward`. You use a function (we say you *call* it) by giving the name, followed by values (called *arguments*) in parentheses `()`.

You can also wrap up a piece of your own code in a function, using the `def` keyword. It might be code you want to use in several places, or even put in a library for someone else to use.

Here's how to define a function to draw a shape with equal sides. You're nearly there already. **Change** your program to look like this:

```
from turtle import *
speed("slowest")

def polygon(a, n):
    for i in range(n):
        forward(a)
        left(360/n)
```

Notice what has happened to *a* and *n*: you deleted the lines that give these variables their values. Instead, *a* and *n* are mentioned in the function definition, and they only exist inside the function while it is working. You give them their

values each time you call the function.

Hint: You type the next few things in the shell, after your program has run. Code you type in the shell is shown with the shell prompt `>>>`. (Don't type the `>>>`.)

Now, if you run your program ... nothing much happens. At the shell prompt (right) try:

```
>>> polygon(50, 3)
>>> polygon(50, 4)
>>> polygon(50, 5)
```

You should get a triangle, a square and a pentagon. (If you got an error message, try to figure out from it where you went wrong.) Now that your shape is a function, you can make a pattern with it. Try:

```
>>> clear()
>>> for i in range(3):
    polygon(100, 6)
    right(120)
```

Try to work out what this does, then run it:

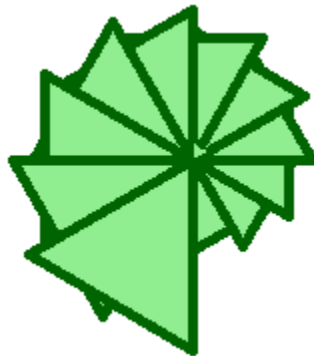
```
>>> clear()
>>> for i in range(20):
    polygon(5*i, 3)
    left(30)
```

2.2.5 A splash of colour

Our thin black lines are a little dull.

What is beautiful?

You've reached the point where simple drawings turn into pleasing shapes. It's worth asking why.



We like symmetry, repetition and precision, which are things computers do well. And we like to discover a pattern, which is what your code supplies. Similarity to things we've seen in nature, and colours that "go together" also make objects look right to us.

One theory is that our human mind is constantly looking for simple explanations for what we see. The pattern has to be complex enough to be interesting, but simple enough to make sense. The programs here and in projects to come *are* quite simple, although you might not guess the program from the picture.

In the little universe of our computer, shapes that resemble nature are called into existence with just a few words.

You can change the pen you use, or fill the shapes you draw with colour. Notice that the `turtle` module uses the American spelling “color”. **Change** your program like this, or with some other shapes, to try out the colour functions:

```
from turtle import *
speed("fastest")

def polygon(a, n):
    for i in range(n):
        forward(a)
        left(360/n)

pencolor("dark green")
pensize(5)

fillcolor("light green")

for i in range(20):
    begin_fill()
    polygon(5*i, 3)
    end_fill()
    left(30)
```

It is easy to guess what most of the new functions do. The bit that may be puzzling you is how to fill the shape with colour. The call to `begin_fill` means “start remembering what you’ve drawn”. Then you can draw any shape (or as many shapes as you like). And finally, the call to `end_fill` says “fill what you remember drawing”.

What difference does it make if `begin_fill` and `end_fill` are outside the loop, like this:

```
begin_fill()
for i in range(20):
    polygon(5*i, 3)
    left(30)
end_fill()
```

2.2.6 Read The Friendly Documentation

Press F1, and IDLE will open the Python documentation in a browser. Go to the “Global Module Index”, then to letter “t”, and the “turtle” module. All the functions we have used and many more are explained. Visit a few you think you understand (like `forward`), then take a look at some you might be interested in trying out.

You won’t understand all of it, but don’t let that put you off. Try things; make mistakes; learn.

2.3 Spectacular Spirals

This project uses turtle graphics to draw spirals: choose your own design. On the way, we meet a spiral that is 200 million years old.

2.3.1 First take your polygon

Start the project by making an empty file `spiral.py`. Right-click and open it with IDLE. Type this (or copy most of it from `final.py` in the introduction) and save it:

```
from turtle import *
speed("fastest")

def polygon(a, n):
    "n-sided polygon, side a."
    for i in range(n):
        forward(a)
        left(360/n)
```

The new line is called a *documentation string*, a little reminder of what the function does. It makes no difference to what the code does, but in IDLE, when you get as far as typing `polygon(`, IDLE will pop up this string as a reminder.

Important: Never throw code away.

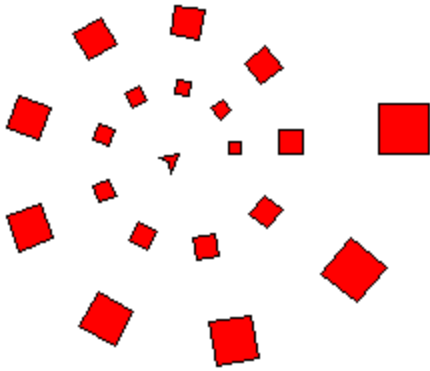
In these projects we explain the program a piece at a time. Mostly you should **add** each new code fragment to the program you have already, normally at the end. Sometimes the fragments show a **change** to what you have.

Occasionally, there is a short fragment of code that you only need for a few tests, then you’ll replace it.

If ever *you* want to delete a lot of code, first use “File” >> “Save Copy As” and give a name like `spiral_2.py`, so you can go back to your program as it was.

2.3.2 Offset and fill the polygon

Your program is going to draw lots of different spirals. Here’s a simple example where you can see what’s going on:



Drawing starts with the smallest square tile, nearest the middle, and continues anti-clockwise, putting down tiles.

Each tile is slightly bigger than the last. Each circuit has the same number of tiles, that you choose, 9 in this case. By the time it gets round once, the tile is twice the original size, and twice as far from the centre. It will go round as many times as you choose.

You will draw each tile with your `polygon` method. To put it in the right place, suppose the turtle is in the centre, and pointing the right way. We need to go forward some distance, then draw the polygon. Then we go back to the centre, ready for next time.

In code, it looks like this:

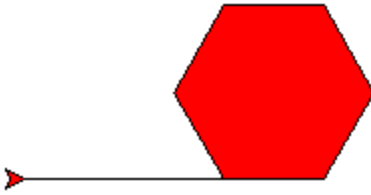
```
def tile(r, a, n):  
    "n-sided polygon, side a, at range r (assumes pen is up)."  
    forward(r)  
    pendown()  
  
    begin_fill()  
    polygon(a, n)  
    end_fill()  
  
    penup()  
    back(r)
```

Add that at the end of your program, and then **add** this to test it:

```
# Test  
fillcolor("red")  
tile(100, 50, 6)
```

Lines that begin with a `#` are comments. Python ignores them: they're a hint to the humans. You should put comments in code you invent, but you don't have to copy them in these examples.

Save and run that. It should look like this:



The initial line is not a fault: it is there because we started with the pen down.

2.3.3 Turn and repeat

Putting the tiles down in a spiral is still too difficult for us in one leap. Think instead how would we put tiles down in a circle.

That would be rather like drawing a polygon, except we use the `tile` function each time instead of `forward`. **Add** after your last function:

```
def circ(r, scale, n, tiles):
    "Circle of n-sided polygons at radius r and size r/scale."
    # Angle between each tile
    step = 360 / tiles

    # Now draw it
    penup()
    for i in range(tiles):
        tile(r, r/scale, n)
        left(step)
```

Change your test code at the end to read:

```
# Test
fillcolor("red")
circ(100, 10, 4, 9)
```

Save and run: you should get a circle of 9 little squares.

2.3.4 Grow a little each time

How can we turn the circle into a spiral? This means making the distance from home (the radius) and the tile size grow each time a tile is placed.

Here “grow” means that we should multiply the size and the radius by some amount each time we put down a tile. The growth factor should be only a little more than one, or the size will get huge in just a few tiles. Remember what happens to powers of numbers from chapter 1?

We have to work out the starting radius and the growth per tile. Choosing that number, a little bigger than one, to get the final size you want, is the the hardest part. You may not have learned the maths for this yet, but the comments explain what’s happening. **Add** after your last function:

```
def spiral(r, n, scale, cycles, m, growth):  
    """Spiral of n-sided polygons out to radius r and size r/scale,  
    in given number of cycles of m steps, growing each cycle.  
    """  
    # Angle between each tile  
    step = 360 / m  
  
    # Total number of tiles (made a whole number)  
    p = int(cycles*m) + 1  
  
    # Growth between each tile  
    g = growth ** (1/m)  
  
    # Starting radius (this will grow with each tile placed)  
    r = r / (growth**cycles)  
  
    # Now draw it  
    penup()  
    for i in range(p):  
        # As the distance from the centre grows, so does the polygon  
        tile(r, r/scale, n)  
        left(step)  
        r = r * g
```

You can see this is like the `circ` function, but with extra code to make the size change. Finally, we're ready to try this out. At the end of your program, **add**:

```
def example():  
    fillcolor("red")  
    spiral(100, 4, 4, 2, 9, 2)
```

The call to `spiral` has a lot of arguments. In order it says that you want:

- a final size of 100.
- squares (4 sides)
- ... that are 4 times smaller than the distance from the centre
- to go round twice
- to have 9 tiles per revolution
- to grow by a factor 2 each revolution

Save and run this. Then at the shell prompt type:

```
>>> example()
```

You should get the example we showed earlier.

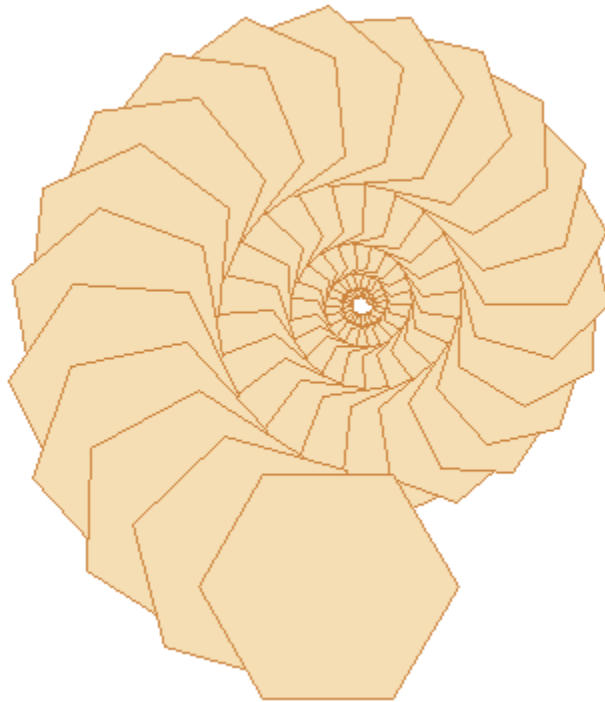
2.3.5 Choose wild numbers

Here is a suggestion to **add**:

```
def ammonite():
    pensize(1)
    color("peru", "wheat")
    spiral(100, 6, 3/2, 4.7, 22.5, 2)
    hideturtle()
```

The Ammonite

This spiral looks like the shell of a creature that lived in Earth's oceans 200 million years ago.



The ammonite lived in the last chamber of a spiral shell, while the others it filled with air. It could only grow in proportion to the existing size of the shell: too big and it would sink and die, because there would not be enough air to keep it afloat.

This exact proportion of the new chamber to the previous ones is the rule built into your program. Your program helps explain why the creature grew in a perfect geometric spiral.

And here is another a suggestion to **add**:

```
def vortex():
    pensize(5)
    color("navy blue", "royal blue")
    spiral(500, 6, 3, 15, 4.2, 1.4)
    hideturtle()
```

Call them at the shell prompt, like you did with `example()`. The function call `reset()` will clear the drawing window between tests.

In the `vortex` you get a sort of two-way spiral because the number of tiles per cycle is not a whole number, but is close to a whole number (4.2). So what might have been 4 straight lines becomes 4 curved lines. What number would make them curve the other way?

Try calling `spiral` in your own function with a variety of numbers. If you find a combination you like, give it a new name.

2.4 Colour in the computer

This project explores how colour is represented as numbers in a computer, by showing the colours you can mix on your screen, from the basic red, green and blue that a monitor can produce. It assumes you know about for-loops and functions. It introduces the idea of tuples to represent colour.

2.4.1 Red, Green, Blue

Start the project by making an empty file `rgb.py`. Right-click and open it with IDLE. Type in the editor:

```
# Explore RGB colour
from turtle import *

def square(a, c):
    fillcolor(c)
    begin_fill()
    for i in range(4):
        forward(a)
        left(90)
    end_fill()
```

Save and run this file. Then in the shell type:

```
>>> square(50, "lime green")
```

You should get a filled square of size 50.



This command, and the code in most other projects in this set, select colours by name. It is difficult to know what names you can use, or what they mean. (Look up “X11 colour names” in Wikipedia if you care about the difference between “light goldenrod” and “papaya whip”.)

Run the program again, just to clear the screen, and type:

```
>>> square(50, (.2, .8, .2) )
```

You should get exactly the same filled square as before.

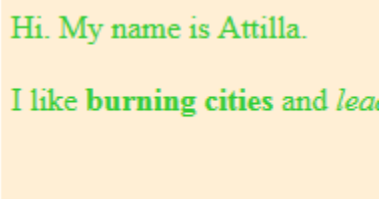
RGB colour in web pages

On web pages, in HTML and CSS, you can use similar names to the ones in the `turtle` module. (Just remove the spaces.)


```

1 body {
2   background: papayawhip;
3 }
4 p {
5   color: limegreen;
6 }

```

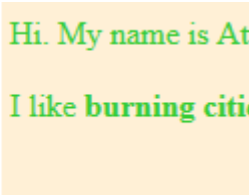


You can also use the notation `rgb(20%, 80%, 20%)` (that's lime green again). This is the method most like the tuples that `turtle` uses.

```

1 body {
2   background: rgb(100%, 94%, 84%);
3 }
4 p {
5   color: rgb(20%, 80%, 20%);
6 }

```



You will most often see colours written in hexadecimal. “lime green” is `#50CD50`. Python `turtle` can understand these hexadecimal colours too, but humans shouldn't have to.

In the `turtle` module, the names for colours are turned into 3 numbers, that can be written in like this `(r, g, b)`. This is called a **tuple** in Python. A **tuple** is a lot like a list, except that you can't change what's in it.

The numbers in a colour tuple have to be between zero and one. They represent the amount of red, green and blue in the colour, in that order. `(1,0,0)` is red, `(0,1,0)` is green, `(0,0,1)` is blue, and `(0,0,0)` is black. Every other colour is a mixture of some kind. We call this the “RGB model”. Try a few mixtures:

```

>>> square(50, (.5, 0, 0))
>>> square(50, (.5, .7, 0))
>>> square(50, (.5, .3, 0))
>>> square(50, (.5, .3, .6))

```

Remember, you don't have to type all this: pull down the previous line with up-arrow and the enter key, then change the parts that should be different.

2.4.2 An RGB colour square

Let's write a program to experiment some more with RGB colours.

We need a program that will step through colour combinations, and show each of them to us. Because there are three colours, this is a little messy, but for two colours we can make a grid of little squares. **Add** this function to your code:

```

def rg_square(a, blue=0, N=10):
    # N+1 little squares wide so ...
    a = a/(N+1)
    start = position()
    for i in range(N+1):
        green = i/N
        # Go to start of row i
        setposition( start + (0,i*a) )
        for j in range(N+1):
            # Draw square j on row i
            red = j/N
            c = (red, green, blue)

```

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```

square(a, c)
forward(a)
setposition(start)

```

And at the end **add** this short program to your code:

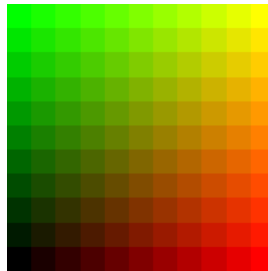
```

# Program
hideturtle()
penup()
speed("fastest")

rg_square(200)

```

Red + green = yellow?!



The design you get from `rg_square(200)` mixes green and red light in different amounts, and the amount of blue is zero. Was it a surprise to find yellow at the top right?

Take a close look at this on your screen. You'll see only green and red dots. Stand back and yellow is what your brain sees. There *is* such a thing as *real, yellow light*, that has a wavelength between red and green. Light waves from a yellow LED or the older kind of streetlamp are 590 billionths of a metre long.

Your eyes have red, green and blue sensors, but they're not very precise. Real yellow light is between red and green, and excites the sensors about equally. So that's all the computer has to do to make you see yellow.

There are infinitely more colours in real life than human eyes can see, or your monitor produce.

There are two loops in `rg_square`, one inside the other. The inner one draws a row of squares with increasing amounts of red. The outer one chooses increasing amounts of green in each row. Save and run. You should see 121 different combinations of red and green.

Finally, let's experiment with adding blue. **Add** this function to your code:

```

from time import sleep

def rgb_cube(a, N=10):
    for b in range(N):
        blue = (b+1)/N
        rg_square(a, blue, N)
        sleep(1)
        update()

```

Change your program to call the new function instead of `rg_square`:

```
# Program
hideturtle()
penup()
bgcolor((0,0,0))
tracer(0)

rgb_cube(200)
```

This begins with the same red-green square as before, then it adds more and more blue (in 10 steps). You can think of these as layers of a cube that has red-green on the bottom, and increasing amounts of blue as you go higher. What you see at the end is the top of the cube. Just for fun, try bigger values of *N*.

2.5 Colour gets arty

This project looks at colour the way a website developer or graphic artist might. It draws a colour wheel using hue and lightness. It makes use of named and default arguments to functions. It is helpful to have done the RGB colour project, but not essential.

2.5.1 RGB and HSL colour

Colour is stored in your computer in the same way that it is put on your screen: as amounts of red, green and blue light. This is called the RGB model. You may already have looked at this in the RGB colour project.

If you are a web designer, or choosing colours for your room, the RGB model is not the best way to think about colour. You might prefer to think in these terms:

- *hue*: the actual colour, where it belongs in the rainbow.
- *saturation*: how strong the colour is; how different it is from grey.
- **lightness**: a scale from black (0), through brightly-coloured (0.5), to white (1).

This is called the HSL model. In this project you will use the Python library module called `colorsys` to convert from HSL to the RGB model that `turtle` needs.

2.5.2 A colour wheel

Start the project by making an empty file `wheel.py`. Right-click and open it with IDLE.

You're going to make a sort of circular rainbow, where the *hue* goes from zero to one. At each hue, you will draw a thin pie-slice (a sector) in that colour. Type in the editor:

```
# Explore HSL colour
from turtle import *
import colorsys

def sector(r, angle, c):
    "Sector filled with colour c"
    fillcolor(c)
    begin_fill()
    forward(r)
```

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```
left(90)
circle(r, angle)
right(90)
backward(r)
end_fill()
```

Save and run this file. Test this function in the shell by typing:

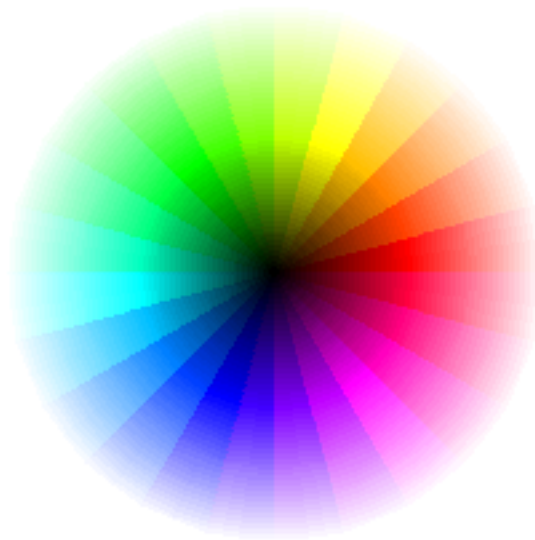
```
>>> sector(200, 15, (0,0,1))
```

You should see a blue sector:



(If it didn't work, debug your code.) In the function call, we specified the colour as an RGB tuple `(0,0,1)`, meaning no red, no green, and 1 full unit of blue. (The numbers in a colour tuple are fractions that run from zero to one.)

Colour in website design



Web designers take a lot of care over the colours they use. One simple way to get a pleasing set of colours is to choose the same hue for all of them.

Here is a set of 5 colours that all have the same hue (0.3), but have different saturation and lightness:



A designer might use the first three for headings, text and background, and the last two for boxes and graphical highlights. The use of a single hue means all the colours go well together. The different saturation and lightness ensure that text can be read against the page background and a small amount of the brightest colour catches the eye.

Now we want to piece together a circle from sectors of different colour. The angle of the sectors has to add up to 360 degrees. **Add** this function to your code after the `sector` function:

```
def wheel(r, sat=1.0, light=0.5, N=24):
    "A colour wheel of radius r with N sectors"
    for i in range(N):
        c = colorsys.hls_to_rgb(i/N, light, sat)
        sector(r, 360/N, c)
```

Notice that the Python library likes to call the HSL model the *HLS* model. (Both are ok, just be careful with the order of the arguments, when you use it.) And when we call it, we have to mention the module `colorsys` in front: this is because we used a different kind of import statement. As a test, save and run, then try `wheel(200)` at the shell prompt.

Add this program at the end of your code:

```
# Program
hideturtle()
penup()
tracer(0)

K = 50
for k in range(K):
    r = 300*(1-k/K)
    wheel(r, light=(K-k-1)/K, sat=1.0, N=120)
    update()
```

`tracer(0)` makes the program run fast by putting off drawing until the call to `update`.

Save and run this. You should see a wheel of all the hues, like the one in the sidebar. *Lightness* runs from zero in the middle (black), to 1 at the rim (white).

To learn how *saturation* affects colour, try altering `sat=1.0` to `sat=0.5`. How would you change the program to display varying saturation, at a lightness you choose? (Remember, a lightness of 0.5 is the most colourful.)

2.6 The Koch Snowflake

This project draws a fractal curve, with only a few lines of turtle graphics code. It assumes you know about for-loops and functions. And it introduces the computer science idea of recursion.

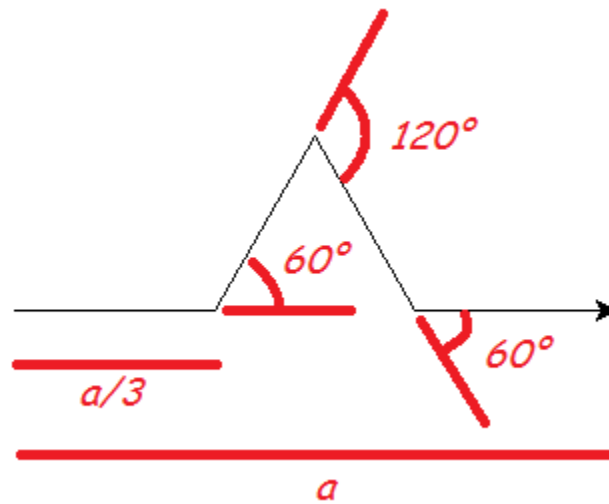
2.6.1 The basic unit

Start the project by making an empty file `koch.py`. Right-click and open it with IDLE.

We're going to define a function that *either* draws a line with a kink in it, *or* draws a straight line the same length. Which one it does will depend on whether the argument `order` is greater than zero. Type this in the editor, save it (ctrl-S) and run it (F5):

A bit of geometry

In the `order=1` case this function draws a carefully-designed curve:



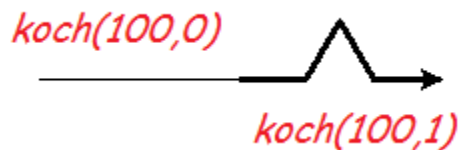
Each of the 4 lines is $a/3$ long, and because of the choice of angles, the distance between the ends of the line is a .

```
# Draw a Koch snowflake
from turtle import *

def koch(a, order):
    if order > 0:
        for t in [60, -120, 60, 0]:
            forward(a/3)
            left(t)
    else:
        forward(a)

# Test
koch(100, 0)
pensize(3)
koch(100, 1)
```

The symbol $>$ means “greater than”. Be careful about the indenting (the spaces before each line). You should get this:



2.6.2 Recursion

This is where your head explodes. You will replace the `forward(a/3)` with another call to `koch`, to draw a kinked line of the same length. That's a line with 4 parts, ...

- and each of those parts has 4 parts,
- and each of those parts has 4 parts,
- and each of those parts has 4 parts, ...

Well, not in practice. In practice, the variable `order` goes down by one each time, and when it hits zero, we just draw a line. **Change** the function `koch`, just a little:

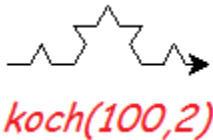
```
for t in [60, -120, 60, 0]:
    koch(a/3, order-1)
    left(t)
```

When a function calls itself, we say it is *recursive*. *Recursion* can solve problems that simple iteration (like a for-loop) cannot.

Save and run. You will get the same two curves as before, because of the test code. But try this in the shell:

```
>>> reset()
>>> koch(100, 2)
```

Now each line segment has become a Koch curve. There are 16 little lines.



What happens for `koch(100, 3)` or `koch(100, 4)`? The order 4 curve has 256 tiny lines. One thing that happens is that it starts to take a long time to draw. A call to `speed("fastest")` before you start drawing will help.

The real Koch Curve

Helge von Koch was a Swedish mathematician. He wanted to prove a curve could exist that was:

- Continuous (you draw it without lifting up your pen).
- Not smooth anywhere (all “corners”).

The real Koch Curve is not any of the things you have drawn. The real Koch curve is what these drawings get closer and closer to as the order goes up, and the lines get smaller. Mathematicians call things defined that way a *limit*.

The curves we draw all have smooth (straight line) segments. But they *look* like the Koch curve, once the straight parts are too small for us to see.



Look at the Koch curve drawing, or snowflake, for order 5 or more. Do you see how the same 3-lobe pattern repeats at different sizes and angles? The whole curve contains miniature versions of itself, smaller and smaller, down into invisibility. This is a property of *fractals*. We say the curve is *self-similar*.

You can easily see how *recursion* makes that happen. In the code we have to stop eventually; in the real Koch curve, it goes on forever.

2.6.3 Need for speed

The drawing is still too slow once `order` is 6 or more. (The calculation is fast.) The `turtle` module wants to show you every change as it is happening: good when debugging, not so good when you just want the result.

We can delay putting the actions on the screen with the `tracer` function. But then we have to tell it to appear using the `update` function, when we're ready.

2.6.4 Looks like snow

The snowflake is made from 3 Koch curves. Although there are only 3 “sides”, it has 6-fold symmetry, like a real snowflake.

Delete the test code from your program, and **add** this program at the end:

```
# Choose colours and size
color("sky blue", "white")
bgcolor("black")
size = 400
order = 0

# Ensure snowflake is centred
penup()
backward(size/1.732)
left(30)
pendown()

# Make it fast
tracer(100)
hideturtle()

begin_fill()

# Three Koch curves
for i in range(3):
    koch(size, order)
    right(120)

end_fill()

# Make the last parts appear
update()
```

Save and run. This should give you a big, white triangle. Adjust `size` so it nicely fills the drawing window when run. Now set `order` to 6 or 7. Run the program and enjoy the result.

How many sides does this have? (Hint: every time the order goes up by one, there are 4 times as many.)

2.7 Celestial Wheels

This project draws a loopy curve, that mathematicians call an epitrochoid, and everyone else calls a spirograph pattern. It introduces the idea of an object and its methods.

2.7.1 Turtles, turtles, turtles

Start the project by making an empty file `loo.py`. Right-click and open it with IDLE. Type in the editor:

```
# Fun with epitrochoids
from turtle import *
```

In the introduction we used only one turtle, and so your commands like `forward` and `left`, went to that unnamed turtle. Here you'll create two other turtles, and send your commands to each of them by name.

Let's see how this works. Save and run your program, so the `import` happens and the shell opens. This is how you make a new turtle, and a variable `ted` to refer to it. Try this in the shell window:

```
>>> ted = Turtle()
```

Now we can give `ted` the turtle some things to do:

```
>>> ted.color("blue")
>>> ted.left(60)
>>> ted.forward(100)
```

These instructions are just like function calls. A function addressed to a particular object is called a *method*. The dot `.` is how you address a *method* to an object in Python. It has to be something the object knows how to do, so:

```
>>> ted.print()
```

raises an error. `print` is a valid *function*, but it is not a *method* of `Turtle` objects.

2.7.2 Setting up the guide turtles

Calculating with positions

A position is described with two numbers (x, y). x is how far it is right of home, and y is how far it is up from home. We call this pair a *vector*. A movement, from one position to another, is a vector too: move x right, and y up.

What was going on when you typed this?

```
>>> mid = (1/2) * (p+q)
```

When you add two vectors, Python knows to add the x -parts together to get the x -part of the result, then the same with the y -parts:

```
>>> p+q
(150.00, 50.00)
```

Subtraction works the same way. You can also multiply a number (called a *scalar*) and a vector together:

```
>>> (1/2) * (p+q)
(75.00, 25.00)
```

Congratulations, you just learned some year 10 maths!

We need two “guide” turtles, each of which draws a circle. Each has to be set up a certain distance from home, so **add** this function to your code:

```
def new_guide(r, c):
    "Return a new guide turtle of radius r and colour c"
    t = Turtle()
    t.speed("fastest")
    t.color(c)
    # Go to (r,0) without drawing
    t.penup()
    t.setposition(r,0)
    t.left(90)
    t.pendown()
    return t
```

Notice this function *returns* a turtle. Save and run, then try this in the shell:

```
>>> g = new_guide(100, "blue")
>>> p = g.position()
>>> g.circle(50, 90)
>>> q = g.position()
>>>
>>> mid = (1/2) * (p+q)
>>> setposition(mid)
```

You should see this:



The unnamed turtle (black) has moved halfway between start and end of the blue arc.

2.7.3 Moving the guides

The first job is to make the guides move in their orbits. **Add** this to your program at the end:

```
def epitrochoid(a, b, L, M=1):
    ta = new_guide(a, "blue")
    tb = new_guide(b, "red")
    # N little steps s make one circle
    N = 500
    s = 360/N
    for i in range(N):
        # ta will go L times round
```

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```

    ta.circle(a, L*s)
    # tb will go M times round
    tb.circle(b, M*s)

# Test
epitrochoid(90, 100, 3, 2)

```

Save and run. You should see blue and red circles drawn.

The blue turtle goes round $L=3$ times, and the red turtle $M=2$ times. You can see how this works in the code. N just has to be a big enough number to make the final curve smooth. N steps of size s make 360 degrees, exactly one circle. So N steps of L or M steps make L or M full circles.

$M=1$ on the first line says that, if you don't give it a value in the call to `epitrochoid`, M will be equal to 1.

2.7.4 Compute the shape

The shape we are looking for is drawn by keeping our pen mid-way between the two guides.

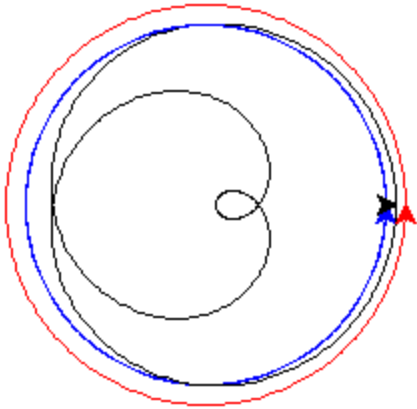
Change the `epitrochoid` function to add these lines:

```

def epitrochoid(a, b, L, M=1):
    "Epitrochoid: a, b are guide radii; L, M the number of orbits."
    ta = new_guide(a, "blue")
    tb = new_guide(b, "red")
    # Local function for midpoint between the guides
    def midpoint():
        return (1/2) * (ta.position() + tb.position())
    # Set start position for unnamed turtle
    penup()
    setposition(midpoint())
    pendown()
    # N little steps s make one circle
    N = 500
    s = 360/N
    for i in range(N):
        # ta will go L times round
        ta.circle(a, L*s)
        # tb will go M times round
        tb.circle(b, M*s)
        # unnamed will be half-way between them
        setposition(midpoint())

```

Save and run. You should see this:



2.7.5 Tidy up

It would be nice if the guide circles were not on the final drawing. **Add** this tidy-up code at the end of `epitrochoid`, and style the unnamed turtle to your liking:

```
def epitrochoid(a, b, L, M=1):
```

```
    ta = new_guide(a, "blue")
```

```
    tb = new_guide(b, "red")
```

```
    .
```

```
    .
```

```
    .
```

```
    # Erase guides
```

```
    guide_erase(ta)
```

```
    guide_erase(tb)
```

```
def guide_erase(t):
```

```
    "Erase what turtle t drew"
```

```
    t.hideturtle()
```

```
    t.clear()
```

```
speed("fastest")
```

```
width(5)
```

```
color("lime green")
```

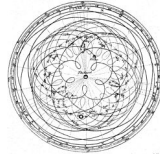
```
# Test
```

```
epitrochoid(90, 100, 3, 2)
```

```
hideturtle()
```

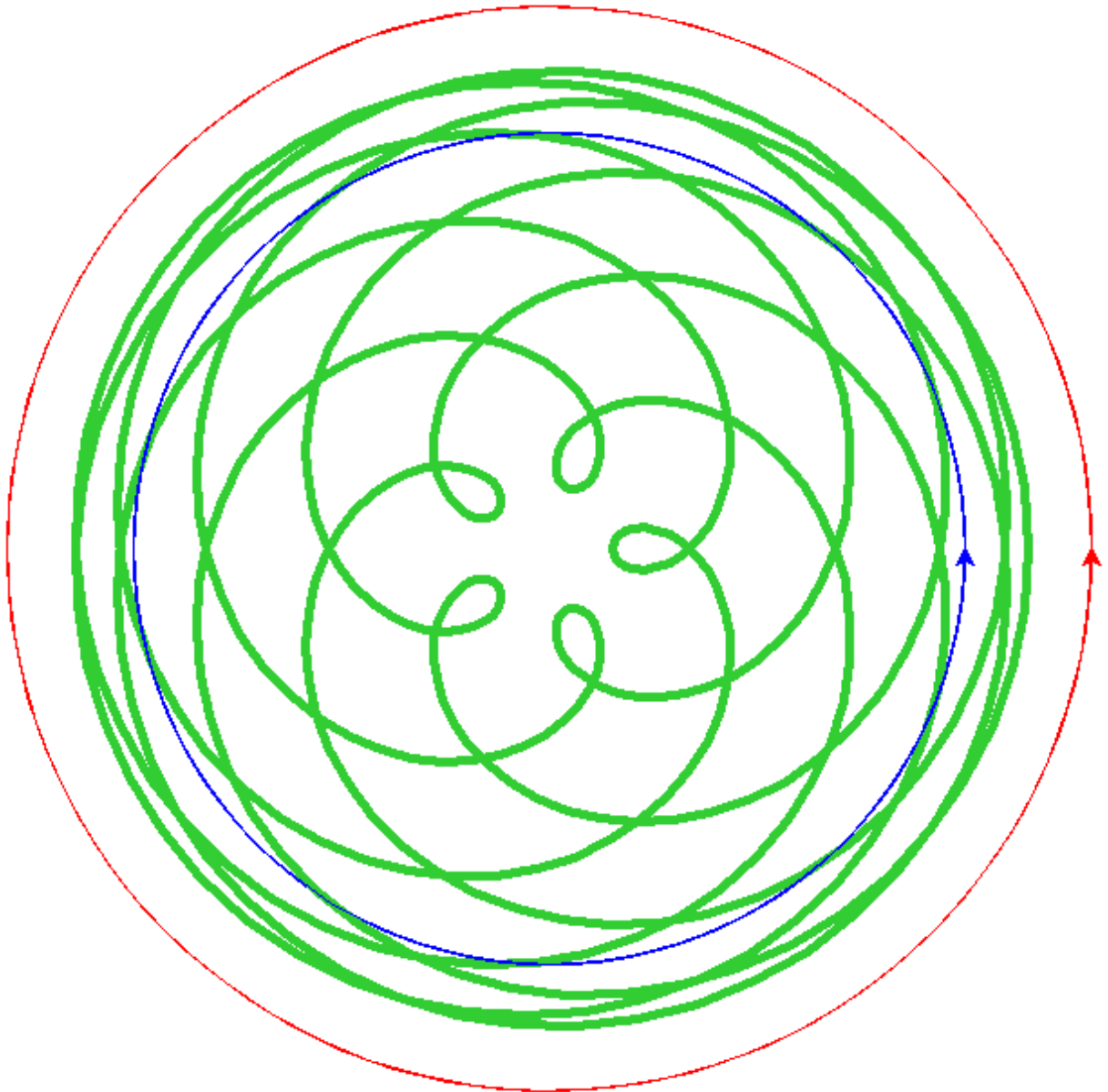
Roman astronomy

Early astronomers took the Earth to be stationary, with the Sun, Moon and planets moving round it. If you take careful measurements of the position of a planet in the sky, you find it speeds up, slows down, and sometimes travels backwards. The Roman astronomer Ptolemy (around AD 145) deduced from this that the planets moved in cycles like the ones you are drawing.



In this theory, the orbit of Venus has the shape you get from:

```
epitrochoid(230, 300, 13, 8)
```



In the 16th century, when the telescope let us see the planets more clearly, we understood that the Earth and Venus both orbit the Sun. The *difference* of these two orbits is the motion Ptolemy observed.

2.7.6 Inspiring examples

Try changing the numbers in the call to `epitrochoid` like this:

```
a, b = 100, 300
epitrochoid(a, b, 4)
color("goldenrod")
epitrochoid(a, b, 5)
color("sienna")
epitrochoid(a, b, 6)
```

(Remember, $M=1$ if you don't give a fourth argument.) Suppose you change just one line now:

```
a, b = -100, 300
```

and run again. When the loops point outwards, the shape is called a hypotrochoid.

What's happening here?

```
a, b = 250, 300
epitrochoid(a, b, 4)
color("goldenrod")
epitrochoid(a, b, 5)
color("sienna")
epitrochoid(a, b, 6)
```

And what about here?

```
L = 6
a, x = 50, 80
epitrochoid(a, L*a, L)
color("goldenrod")
epitrochoid(a, L*a + x, L)
color("sienna")
epitrochoid(a, L*a - x, L)
```

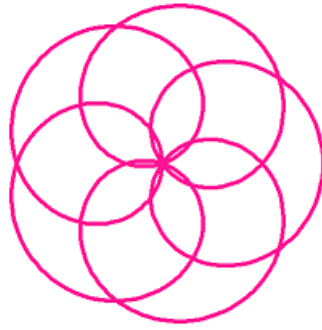
Find other interesting shapes of your own.

2.7.7 Some advanced questions

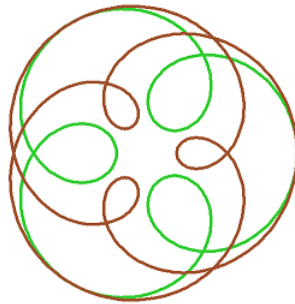
If you like investigating mathematical patterns, this code project poses some interesting questions.

- What determines the number of loops?
- What values for `a` and `b` make the curve pass through (0,0)? (Hint: where would the guide turtles be at that moment?)

A shape in this family, where the curve passes through zero, is called a “rose”.



- When do the loops become points?
- Both curves below have 3 loops: what is the difference between them?



INDICES AND TABLES

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